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An analytical model for prediction of wind fields in tornado-like vortices after touch-down stage

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SUMMARY:
An analytical model is proposed to predict the wind fields in tornado-like vortices after touch-down stage. The radial and vertical velocities are modeled from the mass conservation and the tangential velocity is derived from the momentum conservation. Three velocity components in the corner region of tornado-like vortices are well explained by the proposed model. The predicted three velocity components show good agreement with those obtained from the numerical simulation using the LES turbulence model.

Keywords: Tornado-like vortices, Wind field model, LES turbulence model

1. INTRODUCTION

Tornadoes-like vortices after touch-down stage show three-dimensional complex flow fields as indicated by Ishihara et al. (2011). The wind field in the corner region (the inner region) is important for structure design. An analytical model is required to clearly explain the physical processes and accurately predict the wind field in tornadoes-like vortices.

The wind field of tornadoes-like vortices has been investigated by laboratory simulators (e.g., Matsui and Tamura (2006)), but measurements of the corner region are limited because it is three-dimensional and close to the ground surface. On the other hand, Ishihara et al. (2011) developed a Ward-type numerical simulator using LES model and reproduced the tornado-like vortices.

For structure design, the tangential velocity model by Rankine (1882) has been widely used. However, it cannot express three-dimensional wind field in the corner region due to lack of model for the radial and vertical velocities. To express the three-dimensional wind field, an analytical model is proposed by Burgers (1948) and Rott (1958). While Burgers-Rott model presents that the advection term in the radial momentum equation is zero, Ishihara et al. (2011) indicated that the increase of tangential velocity in the corner region is caused by this advection term. However, a model to predict the radial and vertical velocities in the corner region has not been proposed yet.

In this study, an analytical model to predict the mean wind field of tornado-like vortices is proposed. The three-dimensional wind fields are obtained by numerical simulations and the dominant terms in the mass and momentum conservations are examined in Section 2. The wind fields in the cyclostrophic balance and the outer regions are derived and used as the boundary conditions for the corner flow in Section 3.1. The three velocity components in the corner region are then derived from the mass and momentum conservation to explain the increase of the tangential velocity in Section 3.2.

2. WIND FIELDS IN TORNADO-LIKE VORTICES

In this study, the numerical simulator built by Liu and Ishihara (2015) is used and six cases with the guide-vane angle Φ from $69.4°$ to $84.4°$ are simulated. Fig. 1 shows schematic view of the flow field in tornado-like vortices. The terms in mass and momentum conservation for time-averaged axisymmetric flow field are plotted in Fig. 2. In the cyclostrophic balance region, the centrifugal force balances with the radial pressure gradient. In the corner region, the advection terms due to the radial and vertical velocities appear. The advection term generated by the vertical velocity disappears in the outer region since the vertical velocity is approximately zero in this region. The dominant terms in mass and momentum conservations are shown in Table 1. In this study, the coordinates and velocities are normalized and used hereafter.

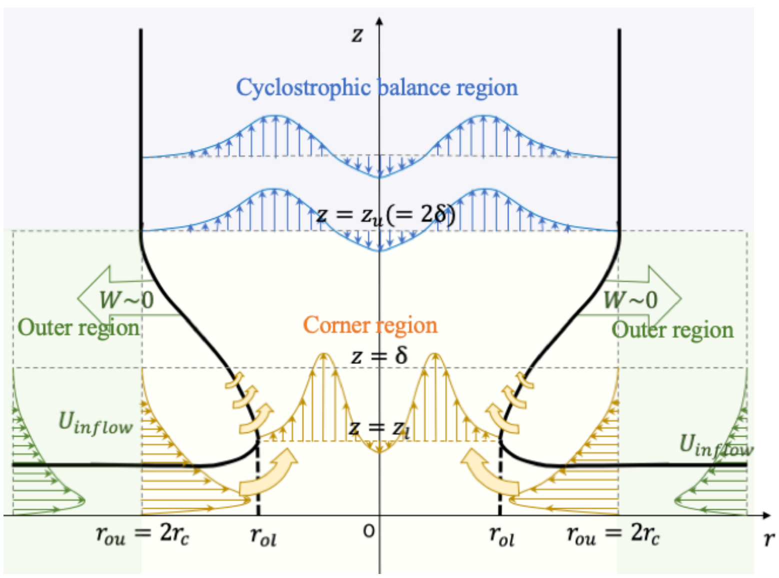
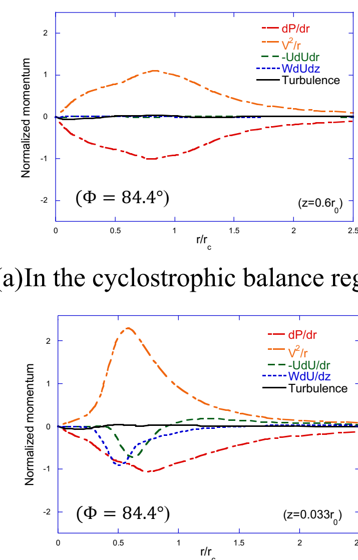


Figure 1. Schematic view of the flow field in tornado-like vortices



(a)In the cyclostrophic balance region

(b) In the boundary layer

Figure 2. Radial momentum balance

Table 1. Dominant terms in mass and momentum conservations

	Mass conservation		Radial momentum balance	
Cyclostrophic balance region	$\frac{\partial W}{\partial z} = 0$	(1)	$-\frac{V^2}{r} = -\frac{\partial P}{\partial r}$	(2)
Outer region	$\frac{1}{r}\frac{\partial (rU)}{\partial r} = 0$	(3)	$U\frac{\partial U}{\partial r} - \frac{V^2}{r} = -\frac{\partial P}{\partial r}$	(4)
Corner region (Inner region)	$\frac{1}{r}\frac{\partial (rU)}{\partial r} + \frac{\partial W}{\partial z} = 0$	(5)	$U\frac{\partial U}{\partial r} + W\frac{\partial U}{\partial z} - \frac{V^2}{r} = -\frac{\partial P}{\partial r}$	(6)

3. ANALYTICAL MODEL FOR THE CORNER FLOW

3.1 Wind profiles at the top and side boundaries of corner region

The radial and vertical velocity components in the cyclostrophic balance and the outer regions are required as the boundary conditions for the corner flow. The vertical velocity component W in the cyclostrophic balance region is obtained from Eq. (1) and the tangential velocity component V is from Eq. (2). The three velocity components at the boundary of the cyclostrophic balance region can be expressed by U_C, V_C and W_C, U_C is zero, V_C and W_C are given by Eq. (7) and Eq. (8). V_C is almost same as that by Burgers-Rott model. The difference is that σ^2 in the proposed model is assumed as a function of the outer circulation $\Gamma_{\infty,c}$, which is an input parameter. W_C is composed of the updraft and downdraft components and is radially distributed as a Gaussian-like shape as shown in Fig. 1. The updraft component is modeled by the axisymmetric average of Gaussian distributions with their center located at $r = 1$. The downdraft component is also modeled by a Gaussian distribution with its center located at the center of vortex. A linear sum of the updraft and downdraft components is taken so that W_C is zero at the center of vortex for simplicity. K is the variance of Gaussian distributions. r_o is the outer core radius as mentioned by Lewellen and Lewellen (2007) as the outer boundary of vertical core flow. Here, r_o is defined by the twice radial location of the updraft peak and is 2 in the cyclostrophic balance region. α is the total flow rate. X is the magnitude of vertical velocity and is set as 1 in the cyclostrophic balance region. $\partial P/\partial r$ is the radial pressure gradient and is assumed as a constant in all regions as mentioned by Ishihara et al. (2011).

$$V_C = \frac{1}{r}\frac{1-\exp\left(-\frac{r^2}{\sigma^2}\right)}{1-\exp\left(-\frac{1}{\sigma^2}\right)}, \sigma^2 = 0.8 \quad [r < 1]; \quad V_C = \frac{1}{r}\left\{\left(1+\frac{\sigma^2}{2}\right) - \frac{\sigma^2}{2}\exp\left(\frac{1-r^2}{\sigma^2}\right)\right\}, \sigma^2 = \Gamma_{\infty,C} - 2 \quad [r > 1] \tag{7}$$

$$W_C = \alpha X \int_0^{2\pi}\left\{\exp\left(-\frac{(r - r_o/2\cos(\theta))^2 + (r_o/2\sin(\theta))^2}{K(r_o/2)^2}\right) - \exp\left(-\frac{r_o{}^2 + (r_o/2)^2}{K(r_o/2)^2}\right)\right\} d\theta \tag{8}$$

$$P_C = \int_\infty^r \frac{V_C^2}{r}\, dr \tag{9}$$

The radial inflow velocity in the outer region U_O expressed by Eq. (10) is proportional to $1/r$, according to Eq. (3). The vertical distribution is approximated using δ and z^*, which are the height at which U_O is zero, and the height of the boundary layer. c_1 is a constant obtained by $\partial U/\partial z(z = z^*) = 0$, and β is the flow rate. W_O is zero and $V_O = V_C$ in the outer region.

$$U_O(r, z) = -\frac{\beta}{r}\frac{(\delta - z)^2}{\delta^3}\left(1 - \exp\left(-c_1\left(\frac{z}{\delta}\right)^2\right)\right) \quad [z < \delta], \qquad 0 \quad [z > \delta] \tag{10}$$

3.2 Wind field in the corner region

The radial and vertical velocity components in the corner region U_I and W_I are simply solved using the mass conservation. The radial distribution of W_I is assumed to be proportional to the core radius $r_o(z)$, as in Lewellen and Lewellen (2007). Considering the boundary conditions with W_C, W_I can be derived as equation (15). The two unknowns are $r_o(z)$ and the magnitude of vertical velocity $X(z)$. In this study, we propose the shape of r_o and derive the distribution of X. r_o is approximated by Eq. (11), which is an increasing from r_{ol} at the ground surface to 2 at the boundary with the cyclostrophic balance region $z = \delta^*$. c_2 is a constant obtained by $dr_o/dz(z = \delta^*) = 0$. $\delta^* = 2\delta$ and $r_{ol} = 1.2$ are adopted for all cases.

$$r_o(z) = 2\left\{\frac{r_{ol}}{2} + \left(1 - \frac{r_{ol}}{2}\right)\frac{\sin\left(\frac{c_2 z}{\delta^*}\right)}{\delta^* \sin(c_2)}\right\} \quad [z < \delta^*]; \qquad r_o(z) = 2 \; [z > \delta^*] \tag{11}$$

By substituting Eq. (16) into Eq. (5), The radial velocity component U_I is derived as a function of $X(z)$ as shown in Eq. (17). The first term represents for the radial outflow expanding the core and becomes negligible in the outer region. The second term is due to the radial inflow and an integral form of mass conservation is applied on a cylindrical volume at the outer region as shown in Eq. (12). The equality between $U(r \gg r_o)$ and U_O derives the differential equation of $X(z)$ as shown in Eq. (13), which is solved as Eq. (14). Here, predicted U_I and W_I by the proposed model agree well with those by CFD as shown in Fig. 3 (b) and (c). Substituting U_I and W_I into Eq. (6), V_I is obtained as Eq. (15) and predicted distributions also show good agreement with those by CFD as shown in Fig. 3 (a).

$$\int_0^r 2\pi r W_I dr = \int_0^z 2\pi r\, U_O dz \tag{12}$$

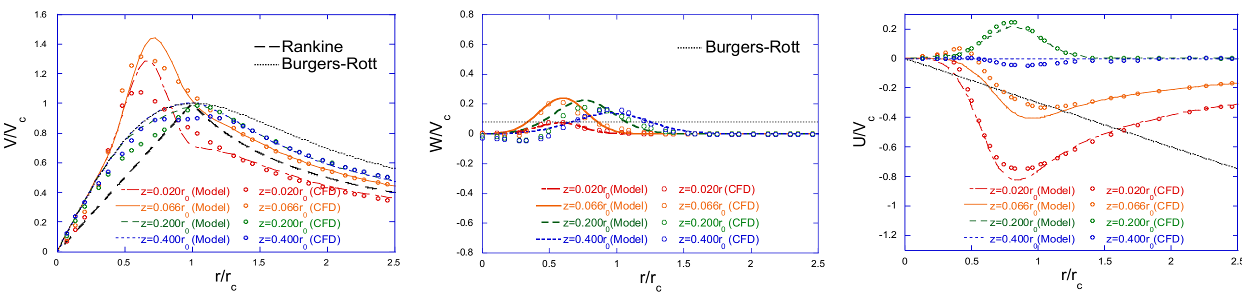
$$\left(\frac{2}{r_o}\frac{dr_o}{dz} + \frac{1}{X}\frac{dX}{dz}\right) = \frac{U_O}{\int_0^z U_O dz} \tag{13}$$

$$X(z) = \frac{1}{(r_o(z)/2)^2}\frac{\int_0^z U_O dz}{\int_0^\delta U_O dz} \tag{14}$$

Table 2. Comparison of the conventional and proposed models

	Rankine model (Rankine, 1882)	Burgers-Rott model (Burgers,1948; Rott, 1958)	Present	
V_I	$V = \begin{cases} V_{max}\frac{r}{r_{V_{max}}} & (r < r_{V_{max}}) \\ V_{max}\frac{r_{V_{max}}}{r} & (r > r_{V_{max}}) \end{cases}$	$V = \frac{1}{r}\frac{1-\exp(-r^2/\sigma^2)}{1-\exp(-1/\sigma^2)}$ $\sigma^2 = 0.8$	$V_I = \sqrt{{V_C}^2 + r\left(U_I\frac{\partial U_I}{\partial r} + W_I\frac{\partial U_I}{\partial z}\right)}$	(15)
W_I	--	$W = 2bz$	$W_I = \alpha X(z)\int_0^{2\pi}\left\{\exp\left(-\frac{\left(r-\frac{r_o}{2}\cos(\theta)\right)^2+\left(\frac{r_o}{2}\sin(\theta)\right)^2}{\kappa\left(\frac{r_o}{2}\right)^2}\right) - \exp\left(-\frac{r^2+\left(\frac{r_o}{2}\right)^2}{\kappa\left(\frac{r_o}{2}\right)^2}\right)\right\}d\theta$	(16)
U_I	--	$U = -br$	$U_I = \frac{1}{r_o}\frac{dr_o}{dz}rW_I - \left(\frac{2}{r_o}\frac{dr_o}{dz} + \frac{1}{X}\frac{dX}{dz}\right)\frac{1}{2\pi r}\int_0^r 2\pi r W_I dr$	(17)

b is an arbitrary constant in Burgers-Rott model



(a) V (b) W (c) U

Figure 3. Comparison of three velocity components predicted by the proposed model and those by CFD

4. CONCLUSIONS

An analytical model to predict three velocity components in the cyclostrophic balance, outer and the corner regions is proposed. The radial and vertical components is analytically derived from mass conservation and the tangential velocity in the corner region is derived by the momentum conservation in the radial direction. The predicted three velocity components show good agreement with those from the numerical simulation using the LES turbulence model.

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